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# Stochastic Prediction of Wire Coupling Interference

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## ABSTRACT

Many EMC analyses of complex systems frequently result in a statement that insufficient knowledge is available to describe accurately the internal relationships of the system's components. This lack of information precludes any rigorous deterministic prediction and, in principle, requires that we express the uncertainties within the model. This paper shows the practical feasibility of the stochastic prediction, as alternative to the deterministic simulation, applied to a class of EMC problems intrinsically affected by randomness. The evaluation of the crosstalk in standard cable bundles, in which several wires are tightly and randomly wrapped together, is the concrete problem that we investigate in this context. We developed a technique based on solving the non uniform MTL for many randomly generated wires' geometries to obtain many crosstalk samples for a single frequency. Finally we validated the method setting up a case study with published experimental results [1,2].

## INTRODUCTION

The prediction of system performances from an EMC viewpoint often entails a high degree of uncertainty. While in principle an arbitrarily accurate analysis of the electromagnetic interactions can be derived by solving the Maxwell's equations with the relevant boundary conditions, the classical deterministic approach may demand more efforts and knowledge than are available even for a relatively simple system. Namely, the increase in detail of the system description corresponds to an increase of the uncertainty of the parameters involved. Nevertheless, many analytical methods currently applied in Electromagnetic Compatibility rely on deterministic models. They usually lead to acceptable prediction errors for the class of problems where the system's interactions and the model parameters can be well defined. This occurs when systems small in electrical size and canonical in geometrical shape are regarded, so that the accuracy and the resolution of the model can be achieved with the maximum extent of details. In real problems the above condition is seldom met, since the interference coupling paths between the system's components involve interaction parameters that are difficult to estimate with accuracy sufficient to permit a meaningful deterministic analysis. Those considerations are particularly enhanced when some system's components exhibit an intrinsic random behaviour that prevents any rigorous deterministic modelisation. In such context, objections to the

significance of a rigorous deterministic approach are legitimate since a single element of the space of the possible outcomes is generated. In conclusion, deterministic models may be valid for the representation of system behaviour on the gross scale but if such models do not readily compare with the observed behaviour of the system then alternate approaches accounting for uncertainties have to be pursued. An attractive methodology consists of defining statistically the system behaviour, i.e. to estimate the probability that some system's variables exceed specified thresholds. This is equivalent to estimate the Probability Density Function (PDF) of the single quantities being investigated once an appropriate statistical description of the uncertain elements of the system is given. The evaluation of the crosstalk in the widely used random cable bundles is the particular EMC problem that will be considered in this paper to illustrate the advantages of the statistical simulation method with respect to the deterministic treatment of the problem. Since the relative positions of the  $n$  wires constituting the random bundles varies unpredictably in any arbitrary cross section along the line axis, any eventual deterministic analysis would not characterise the coupling adequately. Therefore the basic idea is to regard the single analysis output data as a random variable and to consider several geometrical configurations to obtain a set of samples (say 500) sufficient for a meaningful statistical treatment. The complication arising from the random cable analysis does not seem to permit an analytical form of the crosstalk PDF, except for very basic problems. A theoretical expression of the crosstalk PDF between two wires is given in [3], with the constraints that the wire were straight and parallel and the frequency range was such that the wires' lengths were electrically short. Paul et al. [1,2] experimentally investigated the variation of a cable crosstalk due to the relative wire position in two different loads' situations, enhancing respectively the inductive and capacitive coupling mechanism. The results showed that in case of capacitive coupling, the range of variations was larger than 20 dB even for frequencies corresponding to a cable length less than  $0.0001 \lambda$ . In case of inductive coupling, the range of variations was contained within less than 6 dB up to frequencies corresponding to a cable length of  $0.1 \lambda$ . Starting from frequencies where the cable length approaches to  $0.25 \lambda$ , the range of the crosstalk variations was large in both the above cases. This paper aims at mimicking the Paul's experiment via a general model based on the continuous non uniform Multiconductor Transmission Lines (MTL) backed by stochastic simulation. We generate several possible geometries of random bundles with a Monte Carlo method, loaded on both the sides by known impedances. By solving the resulting MTL

equations we produce, for a single frequency, a set of data describing the range of the crosstalk magnitude in a selected location. The systematic statistical manipulation of the obtained data yields the characterisation of the crosstalk in terms of PDF. The wealth in details of [1,2] has made possible to set up a realistic model of the case study therein presented in order to compare the simulation with the experiment.

#### REPRESENTATION OF THE GEOMETRY RANDOMNESS

A random bundle can be defined as a group of  $n$  wires tightly wrapped, their relative position being unknown in any arbitrary cross section along the cable axis. Consider a random bundle parallel to a ground plane and located at an average height  $h$ . Let  $r_i$  and  $R_i$  be respectively the radius of the core and the radius of the wire including the dielectric insulation of the  $i$ -th wire ( $i=1, \dots, n$ ). The constraint on the parallelism of the bundle with respect to the ground plane does not limit the generality of the following method since the relevant theory can be applied also when  $h$  is function of the longitudinal coordinate. To characterise the geometry randomness, we take a single and fixed cross section of the cable containing information on the relative wire positions accounting also for the thickness of the dielectric insulation. Along the cable length, we generate  $M$  identical cross sections with the wires' tags randomly assigned as shown in fig.1.

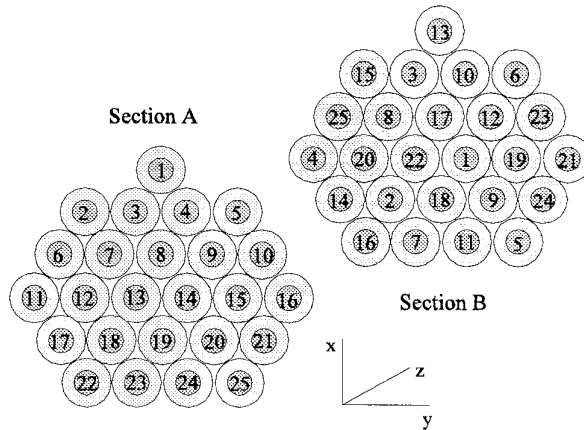


Fig. 1 - Example of wire's tags permutation in two generic cross sections A and B of the random cable.

We assume further that the transversal dimensions of the generic section are much less than  $\lambda$  in order to permit the analysis with the MTL theory. Evidently, this representation contains some approximations. Namely, we hypothesise that the morphology of the overall cable sections is invariant along the longitudinal direction and that in the selected sections all the wires exhibit circular cross section. In most practical cases, the above simplifications are slight alterations of the random cables' real behaviour. In fact when the wires are very densely packed, two arbitrary cross sections of the random bundle appear very alike so that a representation with a higher degree of the detail would produce only a negligible added gain in the model accuracy. The key point of the stochastic model is to describe the variation of the mutual distances between the wires and the random permutations of the wires' identifiers account for it.

Furthermore we note that the path of a single wire in a bundle is smooth in shape and stretched over the longitudinal direction. Considering the above, if the local slope of the bundle with respect to its longitudinal coordinate is small, the assumption of the circular intersection of a single wire with the selected cross sections is also close to the reality. Intuitively, the amount of randomness in a bundle is inversely proportional to its average twist length. This quantity is proportional to the bundle diameter and their ratio is a coefficient  $K$  ranging from 11 to 16 depending on both the mechanical characteristic of the insulating material and the wire gauge. In our representation the number  $M$  of the sampled sections describes the degree of randomness of the bundle and its unequivocal definition is an important element for a successful model. We found empirically that a reasonable behaviour of a single wire path can be obtained by interpolating five equispaced points along an average twist length. Therefore we derived the relation between  $M$ , the length  $L$  of the bundle and its average diameter  $D$ :

$$M = E \left( \frac{4L}{KD} \right) \quad (1)$$

The initial and the final sections are included in the  $M$  selected ones.

#### MTL EQUATIONS AND NUMERICAL SOLUTION

By using the MTL formulation we implicitly maintain that the cable supports a TEM mode. This imposes some limitations on the applicability of the MTL models that in many real situations are satisfied. In particular, by admitting large conductor conductivities, electrically small cross sectional dimensions and neglecting in first approximation the influence of the wires' insulation material on the distribution of the transversal fields, a nearly degenerate TEM mode can be assumed. The above restriction are mostly fulfilled by our geometrical representation but the non uniformity of the resulting MTL introduces additional questions on the validity of the TEM assumption that will be verified a posteriori. In this case the coefficients of the MTL equation depend on the longitudinal coordinate  $z$ . Consider the homogeneous non uniform MTL equation:

$$-\frac{d}{dz} \begin{bmatrix} \mathbf{V}(\omega, z) \\ \mathbf{I}(\omega, z) \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{R} + j\omega \mathbf{L}(z) \\ \mathbf{G}(z) + j\omega \mathbf{C}(z) & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}(\omega, z) \\ \mathbf{I}(\omega, z) \end{bmatrix} \quad (2)$$

where  $\mathbf{R}$ ,  $\mathbf{L}(z)$ ,  $\mathbf{C}(z)$ ,  $\mathbf{G}(z)$  are arrays of dimension  $n \times n$ ,  $\mathbf{V}(\omega, z)$  and  $\mathbf{I}(\omega, z)$  are vectors of dimension  $n$  and  $z \in (0, L)$ . With reference to a coordinate system where the ground plane is located onto the  $x=0$  plane and the projection of the bundle on it is directed along the  $z$  direction, we denote  $(x, y)_i^k$  the center of the  $i$ -th wire of the  $k$ -th sampled section ( $i=1, \dots, n$  and  $k=1, \dots, M$ ). Neglecting the proximity effect between each conductor, we evaluate the element of the  $\mathbf{L}$  matrix at the  $k$  selected sections with the formula:

solution. Given a starting mesh, if the truncation error in one or more mesh intervals is larger than a specified threshold then the affected intervals are automatically bisected until convergence is achieved. This procedure generates a supplementary cost in terms of computer time, but it is justified by the added reliability in the numerical results.

### VALIDATION OF THE THEORY

The previous theory has been applied to the published case study [1,2], to which the reader is referred for more details. A 4 m long bundle composed of 25 AWG #22 stranded wires with PVC insulation was considered. It was located at an average height of 24 mm above the ground plane with the loads connected as shown in fig. 3. This cable was rewrapped many times and the voltage

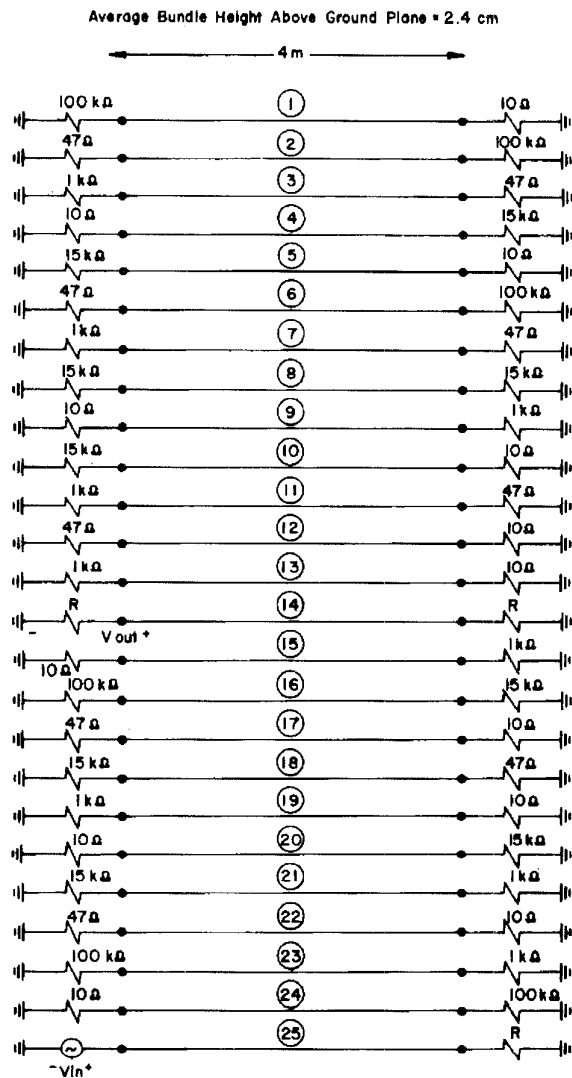


Fig. 3 - Description of the case study [1,2].

transfer ratio  $|V_{out}(f)/V_{in}(f)|$  was measured in each resulting configuration for both  $R=1\text{ K}\Omega$  and  $R=50\text{ }\Omega$  over the frequency range  $10^3 \div 10^8\text{ Hz}$ . Since the average characteristic impedance of a single wire of the bundle is about  $300\text{ }\Omega$ , the cases  $R=1\text{ K}\Omega$  and  $R=50\text{ }\Omega$  enhance respectively the capacitive and inductive coupling mechanism [7]. The core radius of the AWG #22 7/30 stranded wire is 0.381 mm and we estimated the thickness of the dielectric insulation to be roughly identical. So the geometry of the generic bundle cross section was established in our model exactly as shown in fig. 1 accounting for the dimensions described above. Furthermore, the application of the (1) with  $K=12$  (justified by the flexibility of the PVC) leads to consider 130 cross sections in the model. In the (2) the conductance matrix  $G(z)$  and the diagonal matrix  $R$  were set to be zero since we assumed nearly perfect conductors and lossless medium. Our simulation regarded 30 frequencies and for each 500 random geometries were generated.

The figures 4 and 5 show the envelope of both the maximum and the minimum crosstalk magnitude obtained by simulation drawn on the results of a set of measurements published in [1]. We observe a very good matching between testing and prediction. In case of capacitive coupling we note that, below 200 KHz, the difference between the upper envelope and the highest reported crosstalk measurement is contained within 1 dB. In reality we would expect a larger difference. This effect can be due to neglecting the dielectric insulation of the wires in the calculation of the matrix  $C(z)$ , as stated in [8] for a similar situation. We also note that the slope of the envelopes is constant and equal to 20 dB/decade for frequencies below 2 MHz and that between 5 and 10 MHz the range of variation reduces, in agreement with the measured data. Concerning the inductive coupling, the simulation confirms that for frequencies where the bundle electrical length does not exceed  $0.1\lambda$  (in our case below 7.5 MHz) the crosstalk magnitude is virtually insensitive to the wires' locations. In the standing wave region, (i.e. above 7.5 MHz) we note that a sharp minimum was measured at 8 MHz and the simulation reveals it at 15 MHz. This result can be due to the different modal velocities leading to a shift of the resonance frequencies with respect to those calculated with a degenerate TEM mode assumption, as experimentally proven in [9]. Also in this case the slope of the envelopes is 20 dB/decade for frequencies below 1 MHz. This is dictated by the theory and it confirms the success of the statistical simulation. In fact for electrically short lines (and this is our case at 2 MHz) both the inductive and capacitive coupling contributions are proportional to the excitation frequency so that their frequency responses increase at a rate of 20 dB/decade [7]. We also note that for frequencies where the cable exceeds  $0.1\lambda$  in electrical length, the sensitivity of the crosstalk variation with respect to the wires' positions is significant in both the investigated cases, reaching in some points a dynamic larger than 40 dB. Finally, the TEM assumption in densely packed non uniform MTL can be considered appropriate in the light of the obtained results, although refinements in the modelling can be achieved if a TEM mode with different modal velocities, rather than the degenerate case, was considered.

$$\begin{cases} L_{ii}^k = \frac{\mu_0}{2\pi} \ln \frac{2x_i^k}{r_i} \\ L_{ij}^k = \frac{\mu_0}{2\pi} \ln \frac{[x_i^k + x_j^k]^2 + [y_i^k - y_j^k]^2}{[x_i^k - x_j^k]^2 + [y_i^k - y_j^k]^2} \end{cases} \quad (3)$$

$r_i$  being the radius of the  $i$ -th wire. Since the path of a single wire is expected to be smooth inside the random bundle, we maintain that the mathematical description of the component  $x_i$  and  $y_i$  with respect to  $z$  would belong at least to the  $C^1_{(0,L)}$  class of functions. In words, this means that at least the continuity of  $x_i$  and  $y_i$  and of their first derivatives with respect to  $z$  must be assured in the interval  $(0,L)$ . This property can also be extended to the element of the  $\mathbf{L}$  matrix since the arguments of the natural logarithm in (3) do not generate singularities by construction. Therefore we can describe the  $\mathbf{L}(z)$  matrix in the interval  $(0,L)$  by means of smooth interpolation functions, regarding the ordered  $\mathbf{L}^k$  matrixes as vectorial knots. In particular we selected the method proposed by Akima [4] that is based on local procedures. It avoids the unnatural wiggles that sometime may affect other interpolation methods. This approach produces  $C^2_{(0,L)}$  functions composed of piecewise polynomials of degree three at most. Assuming that the cable supports a nearly degenerate TEM mode, the relevant capacitance and conductance matrixes are respectively by:

$$\begin{cases} \mathbf{C}(z) = \mu_0 \epsilon_0 \mathbf{L}^{-1}(z) \\ \mathbf{G}(z) = \frac{\sigma_m}{\epsilon_0} \mathbf{C}(z) \end{cases} \quad (4)$$

$\sigma_m$  being the conductivity of the medium that surrounds the conductors. Without loss of generality, we limit this investigation to the particular load configuration shown in fig. 2.

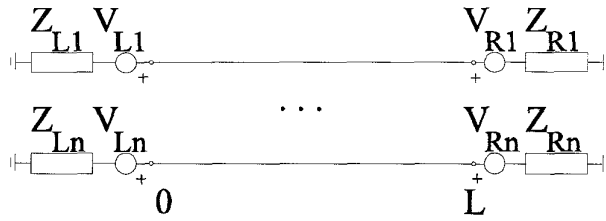


Fig. 2 - Structure of the load configuration under investigation.

Considering that the positive convention for the current is from 0 to L, the above configuration yields the following boundary conditions in matrix form:

$$\begin{bmatrix} 1 & 0 & \dots & 0 & Z_{L1} & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 & 0 & Z_{L2} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & 0 & 0 & \dots & Z_{Ln} \end{bmatrix} \begin{bmatrix} \mathbf{V}(\omega, 0) \\ \mathbf{I}(\omega, 0) \end{bmatrix} = \begin{bmatrix} \mathbf{V}_{L1} \\ \mathbf{V}_{L2} \\ \dots \\ \mathbf{V}_{Ln} \end{bmatrix} \quad (5)$$

and

$$\begin{bmatrix} 1 & 0 & \dots & 0 & -Z_{R1} & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 & 0 & -Z_{R2} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & 0 & 0 & \dots & -Z_{Rn} \end{bmatrix} \begin{bmatrix} \mathbf{V}(\omega, L) \\ \mathbf{I}(\omega, L) \end{bmatrix} = \begin{bmatrix} \mathbf{V}_{R1} \\ \mathbf{V}_{R2} \\ \dots \\ \mathbf{V}_{Rn} \end{bmatrix} \quad (6)$$

The (2), (5) and (6) form a boundary value problem that we solve numerically with a first order finite difference method over a mesh  $0=z_1 < z_2 < \dots < z_{N+1}=L$  using a trapezoidal scheme [5]. Letting  $h_i = z_{i+1} - z_i$ , the boundary value problem can be approximated by the following  $2n(N+1) \times 2n(N+1)$  system of complex linear equation:

$$\begin{bmatrix} \mathbf{B}_0 & 0 & 0 & 0 & 0 & 0 \\ \mathbf{S}_1 & \mathbf{Q}_1 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{S}_2 & \mathbf{Q}_2 & 0 & \vdots & 0 \\ \vdots & 0 & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 0 & \mathbf{S}_N & \mathbf{Q}_N \\ 0 & 0 & 0 & 0 & 0 & \mathbf{B}_L \end{bmatrix} \begin{bmatrix} \mathbf{V}(\omega, z_1) \\ \mathbf{I}(\omega, z_1) \\ \vdots \\ \mathbf{V}(\omega, z_{N+1}) \\ \mathbf{I}(\omega, z_{N+1}) \end{bmatrix} = \begin{bmatrix} \mathbf{d}_0 \\ 0 \\ \vdots \\ 0 \\ \mathbf{d}_L \end{bmatrix} \quad (7)$$

where:

$$\mathbf{Q}_i = \begin{bmatrix} h_i^{-1} \mathbf{I} & \frac{\mathbf{R} + j\omega \mathbf{L}(z_{i+1})}{2} \\ \frac{\mathbf{G}(z_{i+1}) + j\omega \mathbf{C}(z_{i+1})}{2} & h_i^{-1} \mathbf{I} \end{bmatrix} \quad (8)$$

$$\mathbf{S}_i = \begin{bmatrix} -h_i^{-1} \mathbf{I} & \frac{\mathbf{R} + j\omega \mathbf{L}(z_i)}{2} \\ \frac{\mathbf{G}(z_i) + j\omega \mathbf{C}(z_i)}{2} & -h_i^{-1} \mathbf{I} \end{bmatrix} \quad (9)$$

$\mathbf{I}$  being the unit matrix of dimension  $n \times n$  and  $i=1, \dots, N$ .  $\mathbf{B}_0$  and  $\mathbf{B}_L$  are respectively the  $n \times 2n$  matrixes at the left member of (5) and (6).  $\mathbf{d}_0$  and  $\mathbf{d}_L$  are respectively the column vectors of dimension  $n$  at the right member of (5) and (6). The linear system (7) is block-bidiagonal and, for the sizes relevant to our particular application, it can be still efficiently solved with a LU decomposition based on Gaussian elimination with row pivoting and successive iterative refinement using the residual evaluation [6]. The truncation error is also estimated in order to control the accuracy of the computed approximation in relation to the true

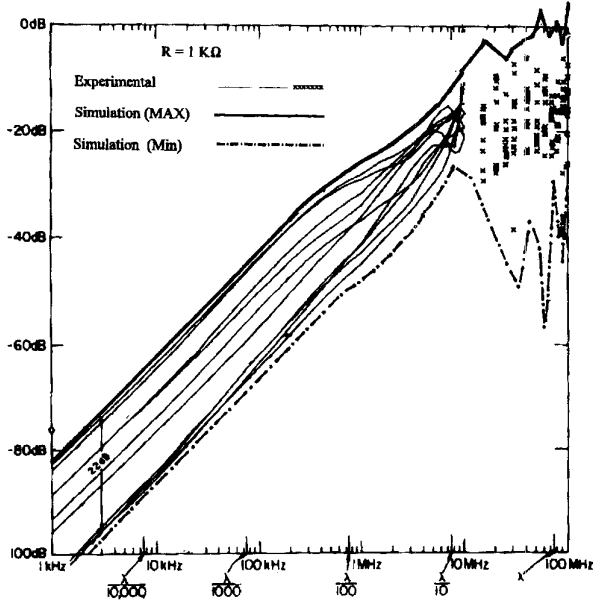


Fig. 4 - Results of the experiment vs. simulation,  $R = 1 \text{ K}\Omega$

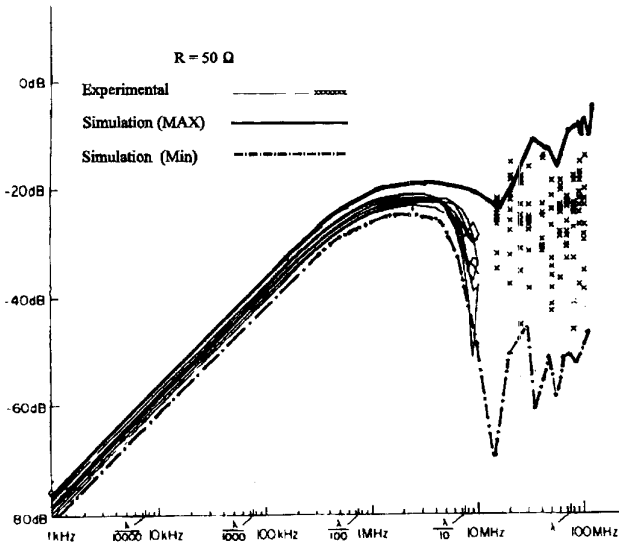


Fig. 5 - Results of the experiment vs. simulation,  $R = 50 \Omega$

### STATISTICAL ANALYSIS

The experiment described in [1] was repeated and additional measurements were performed [2]. Experimental results regarding selected frequencies were reported in tables, allowing us to build and to compare the Cumulative Distribution Functions (CDF) obtained by measurements (60 samples) and simulation (500 samples). The relevant outcomes are shown in figures 6 and 7.

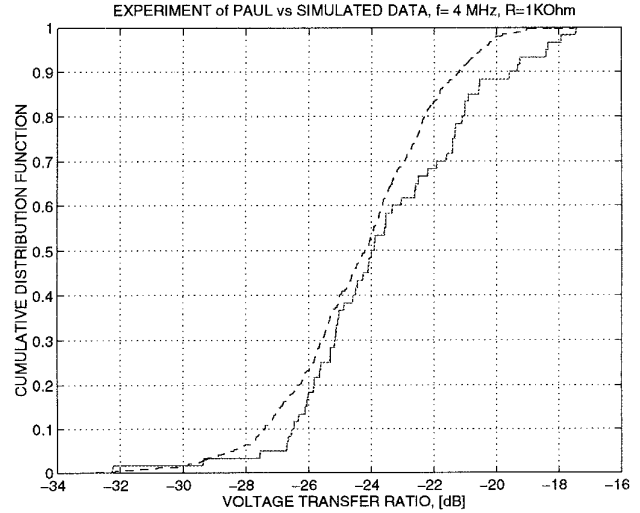


Fig. 6 - Comparison of the CDFs of experimental and simulated data;  $f = 4 \text{ MHz}$ , Capacitive Coupling.

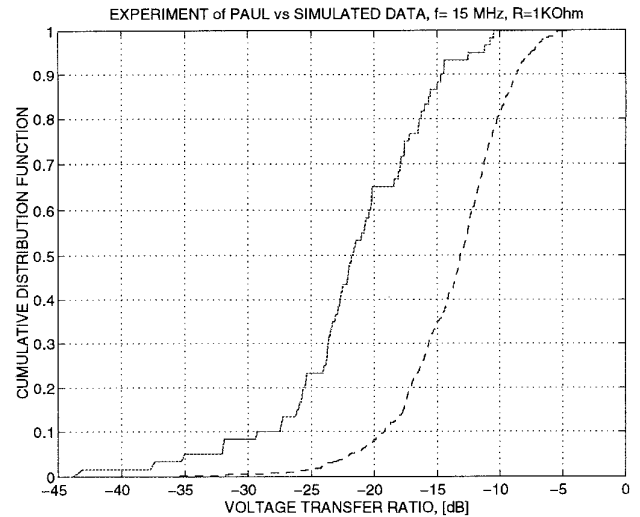


Fig. 7 - Comparison of the CDFs of experimental and simulated data;  $f = 15 \text{ MHz}$ , Capacitive Coupling.

We note a good agreement for the 4 MHz case, that corresponds to a bundle electrical length of  $0.053\lambda$ . The hypothesis that the two data sets are extracted from the same PDF is not rejected with probability 0.95 as it results from the Smirnov Kolmogorov goodness of fit test. In the standing wave region, the situation is different. We observe that the two CDFs are very alike in shape and shifted of about 7.5 dB each other. This enforces the conclusions drawn in the previous paragraph, namely that the assumption of a degenerate TEM mode is valid for frequencies where the bundle length is electrically short. In the standing wave region (i.e.  $L > 0.1\lambda$ ), this assumption yields poorer results but the stochastic simulation describes very well the trend of the CDF. Since even the uniform MTL exhibits the same features as above when a degenerate TEM mode is assumed [10], the legitimacy of

the TEM approach can generally be maintained for this specific type of non uniform MTL. A TEM mode assumption with different modal velocities may provide better results for electrically long lines. This implies that the calculation of the matrixes  $C(z)$  and  $L(z)$  is to be performed accounting for the dielectric insulation of the wires.

The fitting of the observations with simple mathematical expressions simplifies the handling of large data sets. This scope was pursued in [11] for the analysis of the data measured in [2], where several candidate distributions were preliminarily selected in the light of the skewness and the kurtosis coefficients obtained by the measured data. The above coefficients give information on the shape of a generic PDF. Standard tests of goodness of fit were then applied in order to verify the statistical hypothesis of the candidate PDFs, whose parameters were chosen such that the relevant mean and variance equalled, respectively, the sample mean and the sample variance. By grouping the observations in form of histograms in order to represent qualitatively the relevant PDF, we noted that those PDFs were approximately unimodal. This property suggests to use the Pearson's system of curve for data fitting [12]. It attempts to fit the observation with a PDF belonging to the class of function defined by the differential equation

$$\frac{df(x)}{dx} = \frac{x-a}{b_2x^2+b_1x+b_0} f(x) \quad (10)$$

where the coefficients are functions of the first four moments of the data and  $x$  is the random variable. We show in fig.8 the difference between the CDF fitted with the Pearson's method and the empirical CDF obtained by the observations. In almost all the cases we found that the observations can be approximated very well by a Beta PDF.

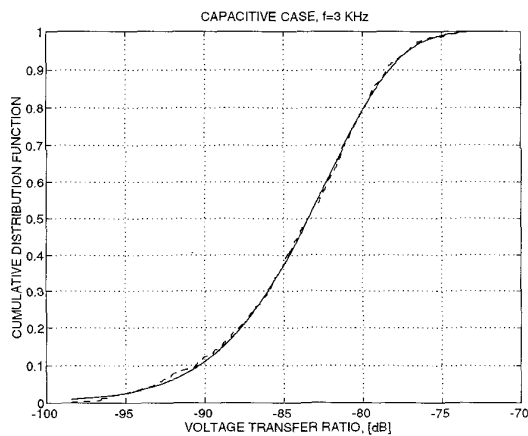


Fig. 8 - CDF of the Capacitive Case,  $f=3$  KHz  
Dotted= Empirical data; Solid= Pearson's curve fitting

The accuracy of the above fitting is a common characteristic for all the investigated cases, that passed the Smirnov Kolmogorov test with significance level of 0.01.

## CONCLUSIONS

This paper has illustrated the feasibility of the stochastic simulation in the EMC discipline. The prediction of a random cable crosstalk has been accomplished by a method based on both the Monte Carlo description of the geometry and the MTL theory applied to the non uniform case. The comparison with published experimental results was quite good and this encourages the application of the stochastic approach to other EMC topics. Furthermore it has been shown that the statistical analysis can provide a deeper insight into the underlying physical process than other methods. The described application has been carried out in a fully numerical fashion on a personal computer, requiring a large response time. However, the involved algorithms can be readily adapted on parallel computers increasing dramatically the efficiency of this technique. The utility of this method is undoubtable since it allows to quantify of the interference risks and to reduce the impact of an indiscriminate overdesign.

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